AP Calc Summer HW - KEY

① a) $\frac{x-4}{x^2-3x-4}$

Factor Denominator!

	x	$+1$
x	x^2	x
-4	$-4x$	-4

$\frac{(x-4)}{(x-4)(x+1)}$

* Don't Lose Numerator!

$= \frac{1}{x+1}$

b) $\frac{x^3-8}{x-2}$

Division Problem! (Tricky, Tricky...)

$$
\begin{array}{r|l}
x-2 & x^2+2x+4 \\
\hline
 & x^3+0x^2+0x-8 \\
 & -(x^3-2x^2) \\
\hline
 & 2x^2+0x \\
 & -(2x^2-4x) \\
\hline
 & 4x-8 \\
 & -(4x-8) \\
\hline
 & 0
\end{array}
$$

$= x^2+2x+4$

c) $\frac{5-x}{x^2-25}$

* Denominator is difference of perfect squares!
$x^2-25 \longrightarrow (x+5)(x-5)$

$\frac{-x+5}{(x+5)(x-5)}$

* Factor $-$ from numerator.

$\frac{-(x-5)}{(x+5)(x-5)}$

$= \frac{-1}{x+5}$

* Don't Lose Numerator!

d) $\frac{x^2-4x-32}{x^2-16}$

$\frac{(x+4)(x-8)}{(x+4)(x-4)}$

$= \boxed{\frac{(x-8)}{(x-4)}}$

* Factor Numerator

	x	-8
x	x^2	-8x
+4	4x	-32

* Denom. is difference of Perfect squares.
$x^2-16 \rightarrow (x+4)(x-4)$

(2) a) Pythagorean Id's
- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$

b) $\cos 2x =$
- $\cos^2 x - \sin^2 x$
- $1 - 2\sin^2 x$
- $2\cos^2 x - 1$

c) $\sin 2x = 2\sin x \cos x$

(3) a) $\frac{1}{x+h} - \frac{1}{x}$

$= \frac{x}{x(x+h)} - \frac{(x+h)}{x(x+h)}$

$= \frac{x-(x+h)}{x(x+h)}$

$= \frac{x-x-h}{x(x+h)}$

$= \boxed{\frac{-h}{x(x+h)}}$

* Need a common denominator!

$\frac{(x)}{(x)}\frac{1}{x+h} - \frac{1}{x}\cdot\frac{(x+h)}{(x+h)}$

Value is 1! ↑ needs 'x' factor

↑ needs 'x+h' factor ↑ value is 1!

b) $\dfrac{\frac{2}{x^2}}{\frac{10}{x^5}}$ ← reads $\frac{2}{x^2}$ divided by $\frac{10}{x^5}$

$\frac{2}{x^2} \div \frac{10}{x^5}$ Dividing fractions = multiply by reciprocal of second term!

$\frac{1\cancel{2}}{1\cancel{x^2}} \cdot \frac{\cancel{x^5}x^3}{\cancel{10}5}$ Check diagonals to reduce GCF.

$= \boxed{\frac{x^3}{5}}$

c) $\dfrac{\frac{1}{3+x} - \frac{1}{3}}{x}$

$= \frac{1}{3+x} - \underbrace{\frac{1}{3}}_{\text{Common Denom!}} \cdot \frac{1}{x}$

Need a common denom.

$\frac{3}{3} \cdot \frac{1}{3+x} - \frac{1}{3} \cdot \frac{3+x}{3+x}$

$= \frac{3}{3(3+x)} - \frac{3+x}{3(3+x)}$

$= \frac{3-(3+x)}{3(3+x)}$

$= \frac{-x}{3(3+x)} \cdot \frac{1}{x}$

$= \frac{-\cancel{x}}{3(3+x)(\cancel{x})}$

$= \boxed{\frac{-1}{3(3+x)} \quad = \frac{-1}{9+3x}}$

(4) a) $4x + 10yz = 0$

$-10yz \quad -10yz$

$\frac{4x}{-10y} = \frac{-10yz}{-10y}$

$= -\frac{2x}{5y} = z$

b) $y^2 + 3yz - 8z - 4x = 0$

$3yz - 8z = -y^2 + 4x$

FACTOR!

$\frac{z(3y-8)}{3y-8} = \frac{-y^2+4x}{3y-8}$

$z = \frac{-y^2+4x}{3y-8}$

* move everything without a 'z' to one side.

(5) a) $\frac{f(x+h) - f(x)}{h}$; $f(x) = x^2 + 2x$

$\frac{(x+h)^2 + 2(x+h) - [x^2+2x]}{h}$

* this is an important idea!!

$= \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$

$= \frac{2xh + h^2 + 2h}{h}$

$= \frac{h(2x + h + 2)}{h}$

$= 2x + h + 2$

b) Binomial Thm! $(x+y)^3$

```
      1
    1   1
  1   2   1
n=3 1   3   3   1
```

$1x^3 + 3x^2y + 3xy^2 + 1y^3$

$= x^3 + 3x^2y + 3xy^2 + y^3$

c) $x^{\frac{3}{2}}\left(x + x^{\frac{5}{2}} - x^2\right)$

$= x^{\frac{3}{2}}\left(x^{\frac{2}{2}} + x^{\frac{5}{2}} - x^{\frac{4}{2}}\right)$

$= x^{\frac{5}{2}} + x^{\frac{8}{2}} - x^{\frac{7}{2}}$

* Need to use exponent rules here. When you multiply, add the exponents!

d) $x = t^2 + 3$

$y = 2t$

So, $t = \frac{y}{2}$

$x = \left(\frac{y}{2}\right)^2 + 3$

$x - 3 = \left(\frac{y}{2}\right)^2$

$\sqrt{x-3} = \frac{y^2}{2^2}$

$\sqrt{(x-3)4} = y$

$y = \sqrt{4x-12}$

(6) a) $\sum_{n=0}^{4} \frac{n^2}{2} = \frac{0^2}{2} + \frac{1^2}{2} + \frac{2^2}{2} + \frac{3^2}{2} + \frac{4^2}{2} = \frac{30}{2} = 15$

b) $\sum_{n=1}^{3} \frac{1}{n^3} = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} = \frac{251}{216}$

(7) a) $\frac{\sqrt{x}}{x} = \frac{x^{\frac{1}{2}}}{x} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$

b) $e^{\ln 3} = 3$

c) $e^{(1+\ln x)} = e^1 \cdot e^{\ln x}$ * Break up exponent using power rules. (exponent)

$= e \cdot x$

d) $\ln 1 = 0$

e) $\ln e^7 = 7$

f) $\log_3 \frac{1}{3} = 3^x = \frac{1}{3} = -1$ * $\log_{base}$ answer = exponent (or use CoB)

g) $e^{3\ln x} = e^{\ln x^3} = x^3$ * log rules!

h) $\frac{14xy^{-2}}{3+2x^{-\frac{1}{3}}y^{-5}} = \frac{x \cdot x^{1/3} \cdot y^5}{3y^2} = \frac{x^{4/3}y^3}{3}$

i) $27^{2/3} = \sqrt[3]{27^2} = 9$ (or $\sqrt[3]{27^2}$)

j) $\frac{3(n+1)!}{5n!} = \frac{3(n+1)\cancel{(n)(n-1)(n-2)\cdots}}{5\cancel{(n)(n-1)(n-2)\cdots}} = \frac{3(n+1)}{5} = \frac{3n+3}{5}$

(8) a) Slope $=-2$, Point $(3,1)$ $= y-4 = -2(x-3)$

b) Contains $(1,-3)$ and $(-5,2)$ $\Rightarrow m = \frac{2-(-3)}{-5-1} = -\frac{5}{6}$

$y+3 = -\frac{5}{6}(x-1)$ or $y-2 = -\frac{5}{6}(x+5)$

c) Slope $=0$, through $(4,2)$ ... Slope $=0$ means horizontal!

$y = 2$

d) Parallel to $2x-3y=7$ and through $(5,1)$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{7}{-3}$

$y = \frac{2}{3}x - \frac{7}{3}$

$y-1 = \frac{2}{3}(x-5)$

e) Perpendicular to 'a' through (3,4)

$m = \frac{1}{2}$ (negative reciprocal of -2)

$y-4 = \frac{1}{2}(x-3)$

(9) Given $v = -2i+5j$ and $w = 3i+4j$

a) $\frac{1}{2}v = \frac{1}{2}(-2i+5j) = -i+\frac{5}{2}j$

b) $w-v = 3i+4j-(-2i+5j) = 5i-j$

c) Length of 'w' $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{9+16} = 5$

(10) Exact Values- You need to know these by ♡!

a) $\sin 0 = 0$ b) $\sin\frac{\pi}{2} = 1$ c) $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

d) $\cos\pi = -1$ e) $\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$ f) $\cos\frac{\pi}{3} = \frac{1}{2}$

g) $\tan\frac{\pi}{4} = 1$ h) $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

(11) a) $y = \sqrt{x-4}$

D: $[4,\infty)$

R: $[0,\infty)$

b) $y = \sqrt{x^2-4}$

D: $(-\infty,-2]\cup[2,\infty)$

R: $[0,\infty)$

c) $y = \sqrt{4-x^2}$

D: $[-2,2]$

R: $[0,2]$

*Don't Forget D: x-axis values, R: y-axis values!

(12) Determine all Points of Intersection!

a) $y = x^2+3x-4$ and $y = 5x+11$

$x^2+3x-4 = 5x+11$

$-5x+11 \quad -5x+11$

$x^2-2x-15 = 0$

$(x-5)(x+3) = 0$

$x=5 \quad x=-3$

	x	-5
x	x^2	$-5x$
$+3$	$3x$	-15

b) $y = \cos x$ and $y = \sin x$ in Q1.

$\frac{\pi}{4}$

(13) Solve for 'x', where 'x' is a real number.

a) $x^2+3x-4=14$

$x^2+3x-18=0$

	x	$+6$
x	x^2	$6x$
-3	$-3x$	-18

$(x+6)(x-3)=0$

$\boxed{x=-6,\ x=3}$

b) $\sqrt{(x-5)^2}=\sqrt{9}$

$x-5=\pm 3$

$x-5=3 \quad x-5=-3$

$\boxed{x=8 \quad x=2}$

c) $2x^2+5x=8$

$2x^2+5x-8=0$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\frac{-5\pm\sqrt{5^2-4(2)(-8)}}{2(2)}$

$=\frac{-5\pm\sqrt{25+64}}{4}$

$\boxed{=\frac{-5\pm\sqrt{89}}{4}} = \boxed{\begin{array}{l}x=1.1085\\ x=-3.6085\end{array}}$ 4 Decimal Places!!

d) $(x+3)(x-3)>0$

$x^2-9>0$

$x^2>9$

$\boxed{x>3 \text{ or } x<-3}$

e) $x^2-2x-15\leq 0$

$(x-5)(x+3)\leq 0$

$x-5\leq 0 \quad x+3\leq 0$

$\boxed{x\leq 5 \quad x\geq -3}$

*watch sign!

f) $12x^2=3x$

$12x^2-3x=0$

$3x(4x-1)=0$

$3x=0 \quad 4x-1=0$

$\boxed{x=0 \quad x=\frac{1}{4}}$

g) $\sin 2x = \sin x \quad [0, 2\pi]$

$\frac{2\sin x\cos x}{\sin x} = \frac{\sin x}{\sin x}$

$2\cos x = 1$

$\frac{2\cos x}{2} = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

h) $|x-3| < 7$

$x-3 < 7 \qquad x-3 > -7$

$x < 10 \qquad x > -4$

i) $(x+1)^2(x-2)^2 + (x+1)(x-2)^2 = 0$

GCF!

$(x+1)(x-2)[(x+1)+(x-2)] = 0$

$(x+1)(x-2)(2x-1) = 0$

$x = -1, \; x = 2, \; x = \frac{1}{2}$

j) $27^{2x} = 9^{x-3}$

$(3^3)^{2x} = (3^2)^{x-3}$ get same base!

$6x = 2x-6$ Power rules! (Exponent)

$-2x \quad -2x$

$4x = -6$

$x = \frac{-6}{4} = -\frac{3}{2}$

k) $e^{3x} = 5$

$3x \ln e = \ln 5$

$3x = \ln 5$

$x = \frac{\ln 5}{3}$

l) $\ln y = 2t-3$

$y = e^{2t-3}$

m) $\log x + \log(x-3) = 1$

$\log x \cdot (x-3) = 1$

$\log (x^2-3x) = 1$

$x^2-3x = 10$

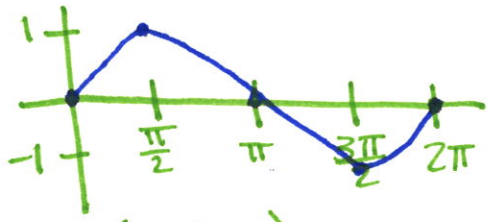
$x^2-3x-10 = 0$

	x	-5
x	x^2	$-5x$
$+2$	$2x$	-10

$(x+2)(x-5) = 0$

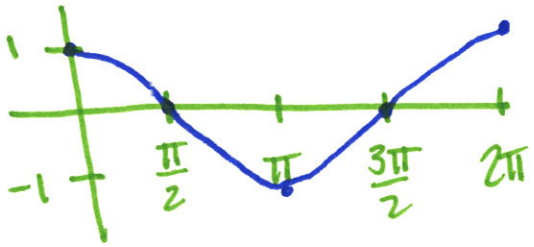
$x = -2, \; x = 5$

(14) a) $y = \sin x$



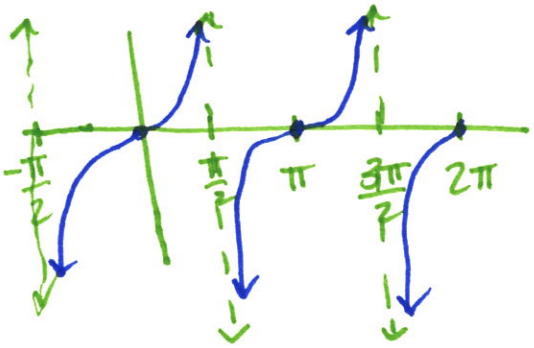
D: $(-\infty, \infty)$
R: $[-1, 1]$

b) $y = \cos x$



D: $(-\infty, \infty)$
R: $[-1, 1]$

c) $y = \tan x$



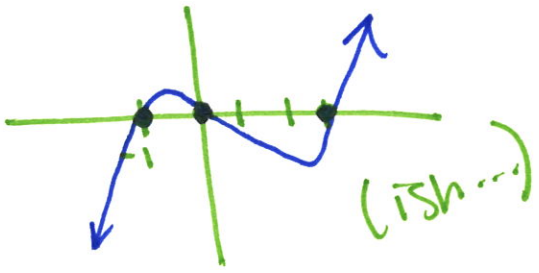
D: $x \in \mathbb{R};\ x \neq \frac{(2n+1)\pi}{2},\ n \in \mathbb{R}$
R: $(-\infty, \infty)$

d) $y = x^3 - 2x^2 - 3x$

$y = x(x^2 - 2x - 3)$ GCF!

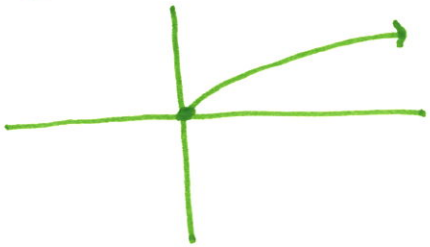
$y = x(x-3)(x+1)$ Factor!

(Roots at $x=0$, $x=3$, $x=-1$)



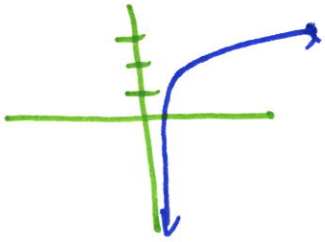
* Check – End behavior – degree 3 so $x \to -\infty$, $y \to -\infty$ and $x \to \infty$, $y \to \infty$.
* Note multiplicity for behavior at roots. odd mult. crosses, even mult. touches!

e) $y = \sqrt{x}$



D: $[0, \infty)$
R: $[0, \infty)$

f) $y = \ln x$



D: $(0, \infty)$
R: $(-\infty, \infty)$

(15) a) zeroes: $x = 0.9088$
$x = 3.8587$
$x = 5.0818$

max: 0.9124 at $x = 2.0739$

b) zeroes: $x = \pm 2.7567$
$x = \pm 3.7097$
$x = \pm 5.505$

max: 0.4158 @ $x = \pm 3.1522$

min: -1.1876 at $x = \pm 4.6586$
-2.3381 at $x = \pm 1.3863$